MTH 230 COMMON FINAL EXAMINATION

Fall 2005

YOUR NAME: ________________________ **INSTRUCTOR:**________________________

INSTRUCTIONS

1. Print your name and your instructor's name on this page using capital letters. Print your name on each page of the exam.
2. This exam consists of this cover page and 10 additional pages containing 10 problems. Be sure your exam is complete before beginning work. Do not separate the pages of this exam.
3. **Show your work**. Work and/or explanation is required on all problems unless otherwise stated; if done well it may result in more credit. Answers accompanied by insufficient, unclear, or incorrect work may receive little or no credit.
4. The points assigned to a problem may not be distributed equally among the parts of a problem.
5. Do not use books, notes, or other references. You may use a TI-82 through TI-86 or equivalent calculator. You are NOT permitted to use calculators capable of symbolic differentiation or integration (such as the TI-89, TI-92, HP-39, or HP-48), portable computers, cell phones, or any other device capable of storing or receiving information.
6. Do not submit scratch paper. Try to solve each problem in the space provided. If you need more space, use the back of this page or other blank space. Be sure to tell on the original page where your additional work can be found, and begin your additional work with the number of the problem being solved.

Problem 1	**40**	
Problem 2	**20**	
Problem 3	**15**	
Problem 4	**20**	
Problem 5	**20**	
Problem 6	**25**	
Problem 7	**15**	
Problem 8	**15**	
Problem 9	**15**	
Problem 10	**15**	
TOTAL	**200**	

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(40) **1**. Evaluate each integral in (a)-(c). Follow the instructions in (d) and (e) on the next page.

(a) $\int \frac{3x^2 - 6\sqrt{x} + 2x\, dx}{x^3}$

(b) $\int 2t\, e^{-3t}\ dt$

(c) $\int_0^{\pi/2} \frac{\sin(\theta)}{(\cos(\theta)+2)^2}\, d\theta$

(d) $\int \frac{x}{x^4 - 1}\, dx$

Write out the form of the partial fraction decomposition of the function, using undetermined constants *A*, *B*, etc. Do NOT attempt to find the values of the coefficients or find the antiderivative.

(e) $\int_0^1 \frac{1}{\sqrt{1-x}}\, dx$

Rewrite this improper integral as a limit, and determine whether it is convergent or divergent. If convergent, find the exact value.

(**20**) **2**. Evaluate each of the following limits, using L'Hospital's Rule where appropriate:

(a) $\lim\limits_{x \to 0} \dfrac{\cos(x) - \cos(2x)}{x^2}$

(b) $\lim\limits_{x \to \infty} x^2 e^{-x}$

(**15**) **3**. A roast turkey is taken from a hot oven and is placed on a table in a room at time $t = 0$. The turkey cools at a rate given by the formula $20\, e^{-0.5t} \, \frac{°\text{F}}{\text{min}}$. Find the total change in the temperature of the turkey during the first ten minutes.

(**20**) **4**. The acceleration function of a particle moving along a straight line is $a(t) = 6t - 9 \frac{\text{meters}}{\text{sec}^2}$. Its initial velocity is $v(0) = -30 \frac{\text{meters}}{\text{sec}}$.

(a) Find the velocity function of the particle.

(b) Use a definite integral to find the displacement of the particle during the time period from 0 to 10 seconds.

(c) Use a definite integral to find the distance traveled during the time period from 0 to 10 seconds.

(**20**) **5**. Let $h(x) = \int_0^x \sin(\sqrt{t})\,dt$.

(a) Evaluate $h'(2)$.

(b) Approximate $h(2)$ using both left-endpoint and midpoint Riemann sums, each with four subintervals.

(**25**) **6**. Consider the region R in the first quadrant of the *xy*-plane, enclosed by the curve $y = -\frac{x^2}{2} + 3$, the line $y = \frac{x}{2}$ and the *y*-axis.

(a) Find the area of R.

(b) Find the volume of the solid obtained from revolving R about the *x*-axis.

(**20**) **7**. A cylindrical tank 20 meters high and 10 meters in diameter sits on the ground. The tank is initially filled with water. Water is pumped out of the tank through a pipe that is 2 meters above the tank's upper rim. How much work is required to empty the tank?

(The density of water in the metric system is 1000 $\frac{kg}{m^3}$.)

(**15**) **8**. Use Simpson's Rule with $n = 4$ to approximate the length of the curve $y = e^{-x^2}$, $0 \le x \le 2$.

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(**15**) **9**. Solve the differential equation $y' = -xy$, $y(0) = 5$. Give your solution as an explicit function $y(x)$.

(15) 10. A bacteria colony grows at an exponential rate, according to the model $P(t) = Ae^{kt}$. Initially, there are 800 bacteria, and after 5 minutes there are 1200 bacteria.

(a) How many bacteria are there after 10 minutes?

(b) How fast is the colony growing after 12 minutes?

(c) When will there be 2000 bacteria in the colony?