

MTH 230 COMMON FINAL EXAMINATION

Winter 2005

YOUR NAME: _____

INSTRUCTOR: _____

INSTRUCTIONS

1. Print your name and your instructor's name on this page using capital letters. Print your name on each page of the exam. Do not separate the pages of this exam.
2. This exam consists of this cover page and 10 additional pages containing 10 problems. Be sure your exam is complete before beginning work. Do not separate the pages of this exam.
3. **Show your work.** Work and/or explanation is required on all problems unless otherwise stated; if done well it may result in more credit. Answers accompanied by insufficient, unclear, or incorrect work may receive little or no credit.
4. The points assigned to a problem may not be distributed equally among the parts of a problem.
5. Do not use books, notes, or other references. You may use a TI-82 through TI-86 or equivalent calculator. You are NOT permitted to use calculators capable of symbolic differentiation or integration (such as the TI-89, TI-92, HP-39, or HP-48), portable computers, cell phones, or any other device capable of storing or receiving information.
6. Do not submit scratch paper. Try to solve each problem in the space provided. If you need more space, use the back of this page or other blank space. Be sure to tell on the original page where your additional work can be found, and begin your additional work with the number of the problem being solved.

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(35) 1. Evaluate each integral. (Part (c) is on page 2.)

(a) $\int t \cos(3t) dt$

(b) $\int \frac{x}{3-2x-x^2} dx$

(c) $\int_0^{\infty} x e^{-x^2} dx$

(Determine whether this improper integral is convergent or divergent. If convergent, find the exact value.)

(15) 2. Evaluate each of the following limits, using L'Hospital's Rule where appropriate:

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

(b) $\lim_{x \rightarrow \pi} \frac{\cos(x) - 1}{x^2}$

- (15) 3. The velocity of an object moving along a straight line is measured every 5 seconds during the interval from 0 to 20 seconds. The results are given in the following table.

t (sec)	0	5	10	15	20
$v(t)$ (ft/sec)	-5	-2	4	11	10

- (a) Give a definite integral which represents the distance traveled by the object during the interval from 0 to 20 seconds. (Hint: “distance traveled” is not the same as “displacement”.)
- (b) Approximate the distance traveled during the interval $[0,20]$ using the Midpoint Rule with two subintervals.
- (c) Approximate the distance traveled during the interval $[0,20]$ using the Simpsons Rule with four subintervals.

- (15) 4. Gasoline is draining out of a tank at a variable rate given by $\frac{3000t}{(3t^2 + 100)^2} \frac{\text{liters}}{\text{minute}}$, where t is the time in minutes. Find the volume of gasoline that drained out of the tank during the interval from 0 to 10 minutes.

(20) 5. Let $g(x) = \int_1^x \sin(t^2) dt$ and $h(x) = \int_1^{x^2} \sin(t^2) dt$.

(a) State *either* Part 1 *or* Part 2 of the Fundamental Theorem of Calculus. Your statement must include the conditions of the theorem.

(b) Find $g'(x)$.

(c) Find $h'(x)$.

(25) 6. Consider the initial value problem $\frac{dy}{dt} = e^{-t}(1 + y^2)$, $y(0) = 0$.

(a) Solve this initial value problem algebraically. Use your solution to find $y(1)$ in decimal form.

(b) Use Euler's Method with step size $h = 0.5$ to approximate the value of $y(1)$. Compare your answer with the value for $y(1)$ found in (a).

- (20) 7. A mining company on Mars wants to know how much work it will take to lift a block of ore with a mass of one metric ton from the surface of Mars to an orbital docking station 200 kilometers above the surface. Use the formula $F = \frac{Gm_1m_2}{r^2}$ for gravitational force to find the work required. In this formula, m_1 and m_2 are the masses of two objects in kilograms, and r is the distance between them in meters. In this problem, r is the distance from the *center* of Mars to the block of ore.

Needed Facts:

*The gravitational constant $G = 6.673 * 10^{-11} \frac{N*m^2}{kg^2}$.*

A metric ton equals 1000 kilograms.

The radius of Mars is 3397 kilometers.

*The mass of Mars is $6.4219 * 10^{23}$ kilograms.*

(30) 8. The region R is bounded by the curves $y = (x - 1)^2$, $x = 0$, and $y = 0$.

(a) Sketch the region R.

(b) Find the area of R.

(c) Give a definite integral for the volume generated when R is revolved about the x-axis. *Do not* evaluate this integral.

(d) Give a definite integral for the volume generated when R is revolved about the line $x = 1$. *Do not* evaluate this integral.

(15) 9. Consider the definite integral $\int_{-1}^1 (x + 2 - |x|) dx$.

(a) Sketch the area represented by this integral.

(b) Evaluate this integral geometrically by interpreting it as an area as in (a).

- (10) 10.** Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Suppose that a cup of coffee initially has a temperature of 95°C , and sits in a room with air temperature of 20°C . After 5 minutes the temperature of the coffee is 80°C .

Write an initial value problem that models the temperature of the coffee as a function of time. By using the information in the last sentence you should be able to find the value of any undetermined constants.