Mathematical Modeling

Example 1

Casey has been selling 70 baseball bats per week at $12 apiece. He plans to lower the price to stimulate sales. He figures that for every $1 reduction in the price, 9 more bats will be sold each week.

a) Find a function that models his weekly revenue in terms of the price of a bat.

b) What will his weekly revenue be if he charges $8 apiece?

Solution:

a) Let x be the price (in dollars) of a bat, and let R(x) be the weekly revenue (in dollars).

Weekly revenue = (price)(number of bats sold in a week).

Number of bats sold = 70 + 9(number of $l reductions in price) = 70 + 9(reduction in price).

Reduction in price = 12 - (price) = 12 - x.

It follows from this that the number of bats sold = 70 + 9(12 - x) = 178 - 9x, so weekly revenue = (price)(178 - 9x) = x(178 - 9x).

Therefore the function R is defined by $R(x) = x(178 - 9x)$.

b) R(8) = 8(106) = $848.

Exercise 2

Casey (from Example 1) can make bats for $2 apiece. Find a function that models his weekly profit in terms of the price of a bat. (Hint: Use the results of Example 1. Most of the work has been done.)

Exercise 3

Art has been selling 80 prints per month at $25 apiece. He has decided that it is time to raise the price. He figures that for every $1 that he increases the price, 2 fewer prints will be sold each month.

a) Find a function that models his monthly revenue in terms of the price of a print.

b) What will his monthly revenue be if he charges $30 apiece?

Example 4

The hypotenuse of a right triangle is three times the length of its shorter leg.

(a) Find a function that models the area of the right triangle in terms of the length of the shorter leg.

(b) If the shorter leg is 5 centimeters long, what is the area of the triangle?

Solution:

(a) Let x be the length of the shorter leg, and let A(x) be the area. Area = $\frac{1}{2}$(base)(height). If we let the shorter leg be the base, then the height would be the length of the longer leg. We know that the length of the hypotenuse is 3x, so we can use the Pythagorean Theorem to find the length of the longer leg.

$$(\text{Height})^2 + x^2 = (3x)^2$$

$$:.\ (\text{Height})^2 + 9x^2 - x^2 = 8x^2$$

$$:.\ \text{Height} = \pm\sqrt{8\text{x}^2} = 2\text{x}\sqrt{2}\ \text{cm}$$ (The height must be positive).

:. The area of the triangle $= \frac{1}{2}(x)\left(2x\sqrt{2}\right) = x^2\sqrt{2}$

Therefore the function A is defined by $A(x) = x^2\sqrt{2}$

(b) $A(5) = 5^2\sqrt{2} = 25\sqrt{2}$ square centimeters.

Exercise 5

A rectangle is four centimeters longer than it is wide.

(a) Find a function that models the area of the rectangle in terms of its length.

(b) If the length is 7 centimeters, what is the area?

(c) Find a function that models the perimeter of the rectangle in terms of its length.

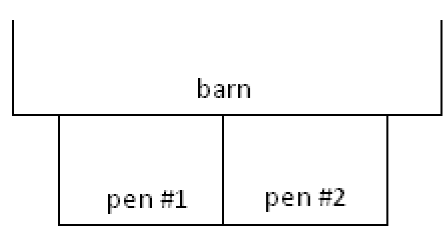
(d) If the length is 7 centimeters, what is the perimeter?

Exercise 6

A rancher wants to build two identical adjacent rectangular pens against the side of his barn, as shown in the picture below. The combined area of the two pens is 1200 square meters.

(a) Find a function that models the amount of fence required to build the pens in terms of the length of fence opposite the barn.

(b) If the length of the side opposite the barn is 40 meters, how much fence will be used?



Example 7

The height of a box is twice the length of its base. The area of the base is 60 square centimeters.

(a) Find a function that models the volume of the box in terms of the length of its base.

(b) If the length of the base is 12 centimeters, what is the volume of the box?

Solution:

If you have trouble getting started, it is often helpful to use a particular number instead of x, and figure out the result for that particular number. Then use the same reasoning with x to find the function.

We will answer part (b) first, then part (a) may seem simpler.

If the length of the base is 12 cm, then the volume = (length)(width)(height) = 12(width)(height).

We need to find the width and height. To find the width, we can use the fact that the area is 60 sq cm.

$(length)(width) = area$, so $12(width)(length) = 60$, and the $width = \frac{60}{12} = 5$ cm.

The height is twice the length $= 2(12) = 24$ cm.

Now we have the volume $= (12)(5)(24) = (60)(24) = 1440$ cu cm.

(a) If the length of the base is x cm, then the volume $= (length)(width)(height) = x(width)(height)$.
We need to find the width and height. To find the width, we can use the fact that the area is 60 sq cm.

$(length)(width) = area$, so $x(width) = 60$, and the width $= \frac{60}{x}$ cm. The height is twice the length $= 2x$ cm.

Now we have the volume $= x\left(\frac{60}{x}\right)(2x) = (60)(2x) = 120$ cu cm.

Therefore the function V is defined by $V(x) = 120x$

(b) Now let us use the function to find the volume if the length of the base is 12 cm.

$V(12) = 120(12) = 1440$ cu cm, as we found earlier.

Exercise 8

Farmer Varner needs to build a rectangular pen next to his driveway. Its area must be 600 square meters. The fence for the driveway side of the pen costs $5 per meter, and the fence for the other three sides costs only $3 per meter.

(a) Find a function that models the cost of the fence in terms of the length of the driveway side.

(b) How much will the pen cost if the driveway side is 30 meters long?

Exercise 9

A box with a square base is to be built for \$96. The material for the sides costs \$6 per square foot, and the material for the top and bottom costs \$8 per square foot.

(a) Find a function that models the volume of the box in terms of the length of its base.

(b) If the length of the base is 2 feet, what is the volume of the box?

Answers to Selected exercises:

6a) $f(x) = x + 3\left(\frac{1200}{x}\right) = \frac{x^2+3600}{x}$, where x is the length of fence opposite the barn, in meters, and $f(x)$ is the amount of fence required to build the pens, also in meters.

6b) $f(40) = 130$ meters

9a) $V(x) = x^2\left(\frac{12-2x^2}{3x}\right) = 4x - \frac{2}{3}x^3$, where x is the length of the base, in feet, and $V(x)$ is the volume of the box, in cubic feet.

9b) $V(2) = \frac{8}{3}$ cubic feet